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Probability Distribution

1. Let X be a random variable with the following probability distribution:

x	-2	3	5
f(x)	0.3	0.2	0.5

Find the standard deviation of X!

- Answer :

$E(x) = \Sigma(x.f(x)) = (-2)(0.3) + (3)(0.2) + (5).(0.5) = 2.5$

$V(x) = \Sigma[x - E(x)\ ]^2.f(x)$

$= (-2-2.5)^2(0.3) + (3-2.5)^2(0.2) + (5-2.5)^2(0.5)$

$= 9.25$

Standard deviation of $X = \sqrt{V(x)} = \sqrt{9.25} = 3.04$

2. Suppose that the probabilities are 0.4, 0.3, 0.2 and 0.1, respectively, that 0,1,2,3 power failures will strike a certain subdivision in any given year. Find the mean and variance of the random variable X representing the number of failures striking this subdivision.
   - Answer :

Mean $= E(x) = \Sigma(x.f(x)) = (0)(0.4) + (1)(0.3) + (2).(0.2) + (3)(0.1) = 1$

Variance $= V(x) = \Sigma[x - E(x)\ ]^2.f(x)$

$= (0-1)^2(0.4) + (1-1)^2(0.3) + (2-1)^2(0.2) + (3-1)^2(0.1)$

$= 0.668$

3. If X be a random variable with the following probability distribution:

x	5	7	9
f(x)	0.5	0.2	0.4

Find the mean!

- Answer :

$E(x) = \Sigma(x.f(x)) = (5)(0.5) + (7)(0.2) + (9).(0.4) = 7.5$

Binomial Distribution

1. According to a study published by a group of University of Massachusetts sociologist, approximately 60% of the Valium users in the state of Massachusetts first took Valium for psychological problems. Find the probability that among the next 8 users interviewed from this state that exactly 3 began taking Valium for psychological problems!
   - Answer:

$P = \langle X \,|\, n, p \rangle = \langle X = 3 | 8, 0.6 \rangle = {}_3^8C.p^3.q^5$

$= \frac{8!}{3!5!}(0.6)^3(1-0.6)^5 = 0.12$

2. In testing a certain kind of truck tire over a rugged terrain, I is found that 25% of the trucks fail to complete the test run without a blowout. Of the next 15 trucks tested, find the probability that
   a). from 3 to 6 have blowouts
   b). fewer than 4 have blowouts
   c). more than 5 have blowouts

   - Answer:

   a). $P(3 \le X \le 6) = \sum_{x=3}^{6} b(x;15,0.25)$

   $= {}^{15}_{3}C(0.25)^3(0.75)^{12} + {}^{15}_{4}C(0.25)^4(0.75)^{11} + {}^{15}_{5}C(0.25)^5(0.75)^{10} + {}^{15}_{6}C(0.25)^6(0.75)^9$

   $= 0.225 + 0.225 + 0.165 + 0.091$

   $= 0.706$

   b). $P(X<4) = \sum_{x=0}^{3} b(x;15,0.25)$

   $= {}^{15}_{0}C(0.25)^0(0.75)^{15} + {}^{15}_{1}C(0.25)^1(0.75)^{14} + {}^{15}_{2}C(0.25)^2(0.75)^{13} + {}^{15}_{3}C(0.25)^3(0.75)^{12}$

   $= 0.4607$

   c). $P(X>5) = 1 - \sum_{x=0}^{5} b(x;15,0.25)$

   $= 1 - 0.851044$

   $= 0.148$

3. The probability that a patient recovers from a delicate heart operation is 0.9. What is the probability that exactly 5 of the next 7 patients having this operation survive?
   - Answer:

   $P(X;n,p) = P(5;7,0.9) = {}^{7}_{5}C.(0.9)^5(0.1)^2 = 0.124$

Hyper geometric Distribution

1. If 7 card are dealt from an ordinary deck of 52 playing cards, what is the probability that:
   a) Exactly 2 of them will be face cards?
   b) At least 1 of them will be a queen?
   - Answer:

   a). $P = \dfrac{{}^{12}_{2}C.{}^{40}_{5}C}{{}^{52}_{7}C} = \dfrac{(66)(658008)}{133784560} = 0.3246$

   b) $P = \dfrac{{}^{4}_{1}C.{}^{48}_{6}C + {}^{4}_{2}C{}^{48}_{5}C + {}^{4}_{3}C{}^{48}_{4}C + {}^{4}_{4}C{}^{48}_{3}C}{{}^{52}_{7}C} = \dfrac{49086048 + 10273824 + 778320 + 17296}{133784560}$

   $= 0.4496$

2. A homeowner plants 6 bulbs selected at random from a box containing 5 tulip bulbs and 4 daffodil bulbs. What is the probability that he planted 2 daffodil bulbs and 4 tulip bulbs?
   - Answer:

$$P\langle x_1, x_2, ...x_k \,|\, a_1, a_2, ..., a_k, N, n\rangle = \frac{({}^{a_1}_{x_1}C)({}^{a_2}_{x_2}C)...({}^{a_k}_{x_k}C)}{{}^{N}_{n}C}$$

$$P\langle 2,4 \,|\, 4,5,9,6\rangle = \frac{({}^{4}_{2}C)({}^{5}_{4}C)}{{}^{9}_{6}C} = \frac{(6)(5)}{84} = \frac{5}{14}$$

3. A foreign student club lists as its members 2 Canadians, 3 Japanese, % Italians, and 2 Germans. If a committee of 4 is selected at random, find the probability that all nationalities are represented!
   - Answer:

$$P\langle x_1, x_2, ...x_k \,|\, a_1, a_2, ..., a_k, N, n\rangle = \frac{({}^{a_1}_{x_1}C)({}^{a_2}_{x_2}C)...({}^{a_k}_{x_k}C)}{{}^{N}_{n}C}$$

$$P\langle 1,1,1,1 \,|\, 2,3,5,2,12,4\rangle = \frac{({}^{2}_{1}C)({}^{3}_{1}C)({}^{5}_{1}C)({}^{2}_{1}C)}{{}^{12}_{4}C} = \frac{(2)(3)(5)(2)}{495} = 0.1212$$

Poisson Distribution

1. The average in a year that people living in a certain city, own dogs are estimated to be 8. Find the probability in a year that people randomly interviewed in that city have the 10 dogs!
   - Answer:

$$P\langle x \,|\, \lambda\rangle = \frac{(\lambda^x)(e^{-\lambda})}{x!}$$

$$P\langle 10 \,|\, 8\rangle = \frac{(8^{10})(e^{-8})}{10!} = 0.09925$$

2. During a laboratory experiment the average number of radioactive particles passing through a counter in 1 millisecond is 3. What is the probability that 5 particles enter the counter in a given millisecond?
   - Answer:

$$P\langle x \,|\, \lambda\rangle = \frac{(\lambda^x)(e^{-\lambda})}{x!}$$

$$P\langle 5 \,|\, 3\rangle = \frac{(3^5)(e^{-3})}{5!} = 0{,}1008$$

3. A certain area of the eastern United States is, on average, hit by 6 hurricanes a year. Find the probabilitt that for a given year that are will be hit by 4 hurricanes!
   - Answer:

$$P\langle x \,|\, \lambda\rangle = \frac{(\lambda^x)(e^{-\lambda})}{x!}$$

$$P\langle 4 \,|\, 6\rangle = \frac{(6^4)(e^{-6})}{4!} = 0.1338$$

Diadaptasi dan diambil dari Soal-Soal Latihan
Probability & Statistics for Engineers and Scientist,
Walpole-Myers-Myers-Ye